Networks (BCNs), Boolean Networks with canalysing functions, etc. [4]. We will indicate on these other classes as we progress in the model and their structural properties. We will first present the basic structure of a Boolean network resulting from the model, and build on it for other specializations.

We present a generally accepted Boolean network model with m inputs, $u \in \mathbf{B}^m$ which, in accordance with internal n states, $x \in \mathbf{B}^n$, drive p outputs, $y \in \mathbf{B}^p$, evolving in time $k = 0, 1, \ldots$ per defined update rules. The temporal and structural relations of these basic variables in the model are described as a collection of update rules that relate state evolution to input, and output evolution to those of state and input.

Naturally the network equations are derived in the form of (1) and (2) which we call as Form-1 representation of Boolean networks. In Form-1 of the model, the system matrices of the network are R and S, defining the relations from w, intrinsically via current values of x and u, to future values of x, and y.

Note that, using the structural relations stated in Lemma 2.3 and 2.4, (1) and (2) can be put into, what we will refer to in the sequel as Form-2,

$$w_x(k+1) = \overline{\bar{M}_x \bar{R} + M_x R}\; P_{w_u} w_x(k) \tag{3}$$

$$y(k) = S\; P_{w_u} w_x(k). \tag{4}$$

where P_{w_u} is a $2^{n+m} \times 2^n$ matrix defined as

$$P_{w_u} = \begin{bmatrix} w_u & 0 & 0 & \ldots & 0 \\ 0 & w_u & 0 & \ldots & 0 \\ 0 & 0 & w_u & \ldots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \ldots & w_u \end{bmatrix}.$$

It is clear that $w = P_{w_u} w_x$.

Taking advantage of Lemma 2.3 and Lemma 2.4, (1) and (2) can also be be written as

$$\bar{w}_x(k+1) = (\bar{M}_x \bar{R} + M_x R)\; \overline{P_x \bar{w}_x(k) + P_u \bar{w}_u(k)} \tag{5}$$

$$y(k) = S\; \overline{P_x \bar{w}_x(k) + P_u \bar{w}_u(k)}. \tag{6}$$

which we will call Form-3.

Still, further manipulation of (1) and (2), using Lemma 2.3 and Lemma 2.4, results in

$$x(k+1) = R\; \overline{P_x (M_x \bar{M}_x) \begin{bmatrix} x(k) \\ \bar{x}(k) \end{bmatrix} + P_u (M_u \bar{M}_u) \begin{bmatrix} u(k) \\ \bar{u}(k) \end{bmatrix}} \tag{7}$$

$$y(k) = S\; \overline{P_x (M_x \bar{M}_x) \begin{bmatrix} x(k) \\ \bar{x}(k) \end{bmatrix} + P_u (M_u \bar{M}_u) \begin{bmatrix} u(k) \\ \bar{u}(k) \end{bmatrix}}. \tag{8}$$

which will be referred to as Form-4 for Boolean networks.

Appearently, the promise of the paper to represent Boolean networks explicitly and uniformly in x and u, and w_x and w_u has been fulfilled with the above statements. Representation in (7) and (8) demonstrate the structure matrices M and P for Boolean networks clearly. Several remarks on the above four forms of Boolean network equations are now in order:

Remark 10. Form-2 (Equations (3) and (4)) is significant in the sense that it implies for each input w_u at time k there is one equation

$$w_x(k+1) = \overline{\bar{M} \bar{Z}_i + M Z_i}\; w_x(k)$$