matrix

$$\begin{bmatrix} E \\ EA \\ EA^2 \\ \vdots \\ EA^{n-1} \end{bmatrix}$$

has rank n, in Boolean conjunctive networks too, we have a very similar result but, instead of a real valued matrix, we have a logical matrix, and a logical equation to solve. Lemma 2.9 has solved this problem with a necessary and sufficient condition which explicitly has the number of cycles l within which the initial state is to be determined. Note that the initial state, as Lemma 2.9 indicates, might not be determined uniquely.

Remark 14. From Lemma 2.9, we can conclude for Boolean conjunctive networks, unlike the linear real valued systems, that they can be controllable and/or observable from certain set of states. We leave the problem of identifying, in terms of system and structure matrices, which states can be observable from which states for further research. We believe that the approach outlined in this paper can lead to the solution of this problem for l, $x(0)$, and $U(0, l-1)$. Note, as the controllability problem may have multiple solutions for u in linear systems, similarly that it may also have multiple solutions for $U(0, l-1)$ in Boolean networks. In contrast, the observability problem requires a unique solution for $x(0)$, however, as Lemma 2.9 implies, the solution of the observability problem in Boolean networks, i.e., determining the initial state $x(0)$ from finite measurements, may not have unique solution, provided that the condition of Lemma 2.9 is satisfied for a solution to exist.

One final remark on observability and controllability of Boolean conjunctive networks is that the criteria for observability and controllability can also be obtained for Form-3 equations. Note that Form-3 equations also cover Boolean networks other than conjunctive ones. In terms of R_x, R_u, S_x, and S_u, we have the following results from (10)-(11) due to Lemma 2.9: The Boolean network described by (10)-(11) is controllable if and only if

$$R_x^l w_x(0) \leq w_x(l) \leq Z\overline{Z^T \bar{w}_x(l)} + R_x^l w_x(0),$$

where $Z = \left[R_x^{l-1} R_u \mid R_x^{l-2} R_u \mid \ldots \mid R_x R_u \mid R_u\right]$, and the required $W_u(0, l-1)$ sequence to steer the state from $w_x(0)$ to $w_x(l)$ in l steps is given by the solution set provided by Lemma 2.9, where

$$W_u(0, l-1) = \begin{bmatrix} w_u(0) \\ w_u(1) \\ w_u(2) \\ \vdots \\ w_u(l-1) \end{bmatrix}.$$

The necessary and sufficient condition for the observability of (10)-(11) can also be derived, using Lemma 2.9 similarly to the above result, and it is omitted here.

4.2.3 Spectral Properties

Spectral properties (eigenvalues and eigenvectors) of Boolean networks have been studied in the literature [9]. It has been reported that the eigenvalues of a Boolean network can be a combination of 0 and roots of 1 [9]. Form-3 of Boolean Equations offer a direct computation for the eigenvalues. Preliminary results of the present paper does not include any results