Since K_w must be a stochastic matrix, the Boolean network with feedback law applied becomes,

$$w_x(k+1) = \overline{\bar{M}_x\bar{R}_v + M_xR_v}\ P_{w_v}w_x(k) \tag{23}$$
$$y(k) = S_v\ P_{w_v}w_x(k). \tag{24}$$

where R_v and S_v have exactly a set of desired selection of columns of R and S in a desired order achieved through the stochastic columns of K_w. Notice that the above feedbacked system must also be a result of considering Form-1 equations with the same feedback, i.e.,

$$x(k+1) = RP_{K_w}\ P_{w_v}w_x(k) \tag{25}$$
$$y(k) = SP_{K_w}\ P_{w_v}w_x(k). \tag{26}$$

Remark 15. The above result of the effect of feedback in Boolean networks is unlike the feedback in continuous time systems, where the feedback changes and more importantly adds different dynamics (eigenvalues) to the system at the desired locations in the complex plane. Whereas it is the Boolean network structure that allows a selection of only the existing columns of system matrices R and S through the feedback matrix K_w. In view of Remark 10, if $q < m$, after the feedback law is applied, instead of 2^m stochastic matrices Z_i's we have 2^q different $2^n \times 2^n$ stochastic matrices, each of which is a subset of the columns of R of the pre-feedback system, however now selected differently for different arrangements of v as the new input. Also note that higher number of new inputs, $q > m$, can also be considered, but, due to the necessity that K_w be a stochastic matrix, no new columns can be assigned into R to achieve different dynamics in the network, if it is so desired. Furthermore, this property of the feedback in Boolean networks is general and it applies to all Boolean networks, not just Boolean conjunctive networks. It is due to the stochastic property of the feedback matrix, K_w, that the desired columns that can be selected are rectricted only to 2^m adjacent column blocks of R and S system matrices. Undoubtedly, the feedback law allows the filtering out of any unwanted dynamics within the network. This is one of the main results of the approach in the present paper regarding the effect of feedback in Boolean networks.

Substituting $u = K_xx + v$ in Form-4, one can get:

$$x(k+1) = (A + CK_x)x(k) + Cv(k)$$
$$y(k) = (E + GK_x)x(k) + Gv(k).$$

Using the relations and structure matrices defined in Lemma 2.3 and Lemma 2.4, Form-4 equations with the feedback law can be re-written in Form-1 form as

$$x(k+1) = ((A + CK_x)L_xQ_x + CL_vQ_v)w(k) \tag{27}$$
$$y(k) = ((E + GK_x)L_xQ_x + GL_vQ_v)w(k). \tag{28}$$

From here, comparing (27) and (28) with (25) and (26) of Form-1, one can arrive at the following equalities

$$RP_{K_w} = (A + CK_x)L_xQ_x + CL_vQ_v, \tag{29}$$
$$SP_{K_w} = (E + GK_x)L_xQ_x + GL_vQ_v, \tag{30}$$

which establish the relationship between K_w and K_x.

Also using the relations and structure matrices defined in Lemma 2.3 and 2.4, on Form-2 network representation with the feedback law $u = K_xx + v$, we obtain,

$$w_u = \overline{M_u(K_xL_xQ_x + L_vQ_v) + \bar{M}_u\overline{K_xL_xQ_x + L_vQ_v}}\ w,$$