B. Finding null and range matrices

Consider the LU decomposition with rook pivoting of a (sparse) matrix A:

$$PAQ = L \times U \quad (28)$$

where P, Q are permutation matrices, L is unit lower triangular, and U is upper triangular in the form

$$U = \begin{bmatrix} U_{11} & U_{12} \\ 0 & 0 \end{bmatrix} \quad (29)$$

where U_{11} is an invertible matrix. Such decomposition is found using the LUSOL library [7].

As a right null matrix, RN, and a right range matrix, RM, we may take

$$RN = Q \begin{bmatrix} U_{11}^{-1} \times U_{12} \\ -I \end{bmatrix} \qquad RM = Q \begin{bmatrix} I \\ 0 \end{bmatrix}$$

Let

$$L = \begin{bmatrix} L_{11} & 0 \\ L_{21} & L_{22} \end{bmatrix} \quad (30)$$

be the partition of the matrix L corresponding to the partition (29), where L_{11}, L_{22} are unit lower triangular matrices. As a left null matrix, LN, and a left range matrix, LM, we may take

$$\begin{aligned} LN &= [L_{21} \times L_{11}^{-1} \quad -I]P \\ LM &= [I \quad 0]P \end{aligned} \quad (31)$$

C. Solution to equation (16)

Consider the equation (16), i.e.

$$LN_{11}D_{12}R_{22} + LN_{12}D_{22}R_{22} = 0$$

Without loss of generality we can assume, that $LN_{12} = Y \times LM_{22}$, where LM_{22} is a left range matrix of D. Additionally, from (13), $R_{22} = RN_{22} \times T_{22}$ for some matrix T_{22}. Therefore the matrix Y solves

$$LN_{11}D_{12}R_{22} + Y \times (LM_{22}D_{22}RN_{22})T_{22} = 0$$

By (22), $LM_{22}D_{22}RN_{22} = \widetilde{D}_{22}$. From the generalized Schur decomposition (23), and since $T_{22} = Z_s^2$ (see 10), we obtain

$$(LM_{22}D_{22}RN_{22})T_{22} = Q_s^2 R_D^2$$

Hence,

$$Y \times Q_s^2 = -LN_{11}D_{12}R_{22} \times (R_D^2)^{-1}$$

and as Y we may take $Y = -LN_{11}D_{12}R_{22} \times (R_D^2)^{-1} \times (Q_s^2)'$. Finally:

$$LN_{12} = -LN_{11}D_{12}R_{22} \times (R_D^2)^{-1}(Q_s^2)'LM_{22}$$

D. Computing the generalized Schur decomposition

The generalized Schur decomposition (7) of a regular matrix pair (A, D) can be computed using Lapack's DGGHRD and DGGES functions. Required eigenvalues reordering can be computed using the Lapack's DTGSEN function (see [2]). We use however more efficient algorithm presented in [1] and [10].

E. Matrix multiplication

Efficiency of a chained matrix multiplication

$$A_1 \times A_2 \times \ldots \times A_k \quad (32)$$

highly depends on order in which multiplication of two matrices is performed. Efficient sequence of matrix multiplications is determined by solving a dynamic programming problem.

VI. Related work

Since Blanchard and Kahn, 1980 [6] a number of alternative approaches for solving linear stochastic optimal control problems have emerged including the Anderson-Moor algorithm [4], the Sims' QZ method [12], the Klein's method [9], the Uhlig's method [13], and others. Numerical experiments conducted in [3] shows, that among methods [4], [12], [9], [13], the Anderson-Moor algorithm is usually the fastest and the most accurate.

All these methods, except the Anderson-Moor algorithm, solve the deterministic part using the generalized Schur decomposition (or the QZ decomposition), similarly as in the basic algorithm presented in section II.

To our knowledge, the algorithms of reduction of size of the eigenvalue problem, presented in section III and the algorithm for solving block triangular problems, presented in section IV are new.

VII. Numerical experiments

In this section we analyze efficiency and stability of presented algorithms applied to a large scale DSGE model. We consider the Memo III model [5] - a large scale multisector dynamic stochastic general equilibrium model that was constructed for the purpose of CO2 reduction policy assessment. This model, when represented in the form (1) has 7906 endogenous variables y and 130 shock variables ε.

In table I we compare the blocked (4) and unblocked (3) algorithm. The column time presents total time measured in seconds required to solve the model using given algorithm. The column r_d reports norms of residuals: $r_d = \| AR + DRP \|_1$, where R, P, form the solution obtained by given algorithm. Coefficient R_d is defined as $R_d = \| |A||R| + |D||R||P| \|_1$. Coefficient r_d / R_d gives insights about the backward stability of the blocked and unblocked algorithm. Performance and accuracy of the unblocked and blocked algorithm

TABLE I.

algorithm	time	r_d	R_d	r_d/R_d
unblocked	2.36	3.39e-13	2.25e2	1.50e-15
blocked	0.647	2.07e-10	2.21e5	9.36e-16

Computation is performed on Intel Core i5-7200U CPU @ 2.50GHz with two physical cores. Peak performance in double precision on a single core is 40 GFLOPS.

Table I shows, that both blocked and unblocked algorithm is able to solve a large optimal control problem in very short time. The blocked algorithm is substantially faster. The accuracy measure r_d/R_d suggests, that both algorithms are numerically stable. However in order to draw definite conclusion, much more detailed analysis is required. In the blocked algorithm, the consider three blocks as in (14), where the second block is the biggest irreducible block in decomposition the (14).

Table II presents a split of total computation cost of the blocked algorithm. Structure of computation cost of the blocked algorithm

TABLE II.