parallel robot as in [27] and [28]. The advantage of this type of controllers is low sensitivity versus parameter variations and disturbances. Sliding mode controller has been used for several applications such as Underwater vehicles [27], Active vehicle suspensions [30], Magnetic levitation [31], DC-DC converters [32] and photovoltaic solar in [33].
This paper is organized as follows. In Section2, the dynamic model of 2-DOF parallel manipulator is formulated in the Cartesian space. In Section3, sliding mode controller based on backstepping approach is developed and applied to the direct dynamic model of robot in Cartesian space the Section4, presents simulation results of the proposed controller. Finally, some conclusions are presented in the closing section.

II. Dynamics modeling of Biglide parallel robot

A. Kinematic and geometric analysis

For the geometric and kinematics modeling of a Biglide parallel manipulator, the following conventions are used according to [12], [27]. The manipulator provides 2DoF of translation on the XY plane, the positioning of end effector is represented by operational variables (x,y) driven by two prismatic active joints (q_1,q_2) in the same X axis.
The operational vector is then written as follow:

$$P=[x \quad y]^T \tag{1}$$

The generalized joint variable vector is:

$$q=[q_1 \quad q_2]^T \tag{2}$$

The mechanism has two constant length struts with moveable foot points Figure 1. Both struts have the same lengtha. The relationship between both coordinate vectors is written with kinematic loop-closure constraints Figure 1:

$$\Phi(P,q)=0, \Phi(P,q)=\begin{pmatrix} (x-q_1)^2+y^2-a^2 \\ (q_2-x)^2+y^2-a^2 \end{pmatrix} \tag{3}$$

The Inverse geometric model (IGM) formula is given by:

$$q=g(P) \tag{4}$$

with

$$g(P)\equiv\begin{pmatrix} x-C(y) \\ x+C(y) \end{pmatrix}, C(y)\equiv\sqrt{a^2-y^2} \tag{5}$$

The direct geometric model (DGM) can be derived from (4):

$$P=g^{-1}(q) \tag{6}$$

with

$$g^{-1}(q)=\begin{pmatrix} \frac{q_1+q_2}{2} \\ \sqrt{a^2-\frac{(q_1+q_2)^2}{4}} \end{pmatrix} \tag{7}$$

The relation between the joint space and the operational space is conveniently described by two Jacobian matrices $J_p(P,q)$ and $J_q(P,q)$ is given as:

$$J_p(P,q)\dot{P}=J_q(P,q)\dot{q} \tag{8}$$

The parallel singularities occur when the Jacobian matrix J_p is rank deficient. The Biglide has two parallel singularities:[12]
• High singularity: $q_1=q_2=x$, the struts are superposed and

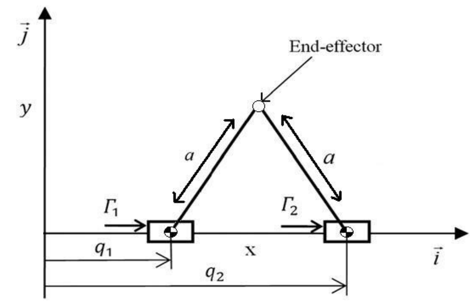

Fig. 1. kinematic schemes of Biglide robot.

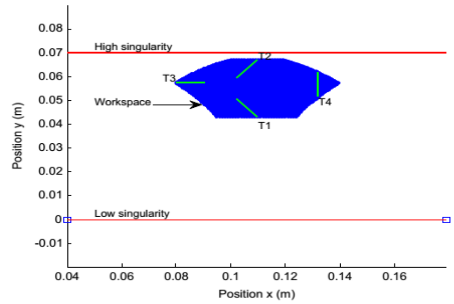

Fig. 2. Workspace and trajectories: $(T1)$ Low trajectory, $(T2)$ High trajectory, $(T3)$ Left trajectory, and $(T4)$ Right trajectory.

$y=0.07$, Figure 2.
• Low singularity: $y=0$, the struts are aligned, Figure 2
The kinematic relationship between end-effector velocities and joint velocities is computed by differentiating (3) with respect to time:

$$J_p(P,q)\dot{P}=J_q(P,q)\dot{q} with J_p(P,q)=\begin{bmatrix} x-q_1 & y \\ x-q_2 & y \end{bmatrix}$$

$$J_p(P,q)=\begin{bmatrix} x-q_1 & 0 \\ 0 & x-q_2 \end{bmatrix} \tag{9}$$

B. Dynamic Model

The dynamics equations of the Biglide in operational space are given as follows:

$$\Gamma=M(P)\ddot{P}+N(P,\dot{P}) \tag{10}$$

with

$P=[x,y]^T$, $M(P)$ is the inertial matrix given as follow:

$$M(P)=\begin{pmatrix} m_1+\frac{1}{2}(m-\lambda_1+\lambda_2) & f_1(P) \\ m_2+\frac{1}{2}(m-\lambda_2+\lambda_1) & f_2(P) \end{pmatrix} \tag{11}$$

with

$$\lambda_{1,2}=ms_{1,2}/a$$

$$\begin{cases} f_1(P)=[(2m_1-3\lambda_1-\lambda_2)y^2+mC(y)^2+J_1 \\ J_2]/(2C(y)\times y) \\ f_2(P)=-[(2m_2-3\lambda_2-\lambda_1)y^2+mC(y)^2+J_1 \\ +J_2]/(2C(y)\times y) \end{cases}$$